Definition V.1

A lesser magnitude $a$ is part of a greater magnitude $b$ if $a$ measures $b$.
magnitude: length, area, volume, angle, positive integer (number)
to measure: to divide

Definition V.2

If $a$ is a part of $b$ (as in def. V.1), we say $b$ is a multiple of $a$.

Definition V.3

A ratio is a sort of relation between two magnitudes $a, b$ of the same kind, denoted
$a : b$ or $\frac{a}{b}$.

Definition V.4

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To have a ratio, a sufficiently large multiple of \( a \) must exceed \( b \), and vice versa (i.e. neither \( a \) nor \( b \) can be infinitely large or small).

Examples:

**Definition V.5**
\[ a : b = c : d \] if for all positive integers \( n, m \), \( na > mb \) implies \( nc > nd \), \( na = mb \) implies \( nc = md \), and \( na < mb \) implies \( nc < md \).

**Definition V.6**
When two ratios are equal, this equality is called a proportion.

**Definition V.7**
If there exists positive integers \( n, m \) so that \( na > mb \) but \( nc \leq md \), then we say \( a : b > c : d \).