MT 453 Elements Day 6

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Proposition I.15

*If two straight lines cut one another, the vertical angles are equal to one another.*

Proof:
Label the angles $\alpha$, $\beta$, $\gamma$, and $\delta$.

We know that $\alpha + \beta = \perp\perp$. (prop. I.13)
But we also know that $\beta + \gamma = \perp\perp$. (prop. I.13)
Thus, $\alpha = \perp\perp - \beta$ and $\gamma = \perp\perp - \beta$. (c.n. 3)
Therefore, $\alpha = \gamma$. (c.n. 1)
Similarly, $\beta = \delta$.

Q.E.F.
Comments:
1. This is the same proof that Euclid gives in the Elements, it just uses different notation which makes it easier to understand.
2. After this proof there is a Porism, which is a consequence of the proof. The Porism was not in the original Elements written by Euclid, but is believed to be included in editions written around 100-200 B.C.
   Porism:
   If two lines cut one another, the angles at the point of section equal four right angles.
This Porism leads to the fact that if you want to surround a point with regular polygons, there are only three ways to do this: with a square, with six equilateral triangles, or with three hexagons. This is a fundamental fact about two-dimensional Euclidean geometry.