Proposition 1.29

If a straight line falls on two parallel lines, then:

1. The alternate angles are equal ($\alpha = \beta$)

2. The exterior angle equals the interior opposite angle ($\delta = \beta$)

3. The interior angles on the same side equal $\perp$ ($\gamma + \beta = \perp$).
Proof:
1. Claim: The alternate angles are equal ($\alpha = \beta$).
   Suppose the alternate angles are not equal; say $\alpha > \beta$.
   Then $\alpha + \gamma > \beta + \gamma$.
   But $\alpha + \gamma = \varnothing$. (prop. I.13)
   So $\beta + \gamma < \varnothing$.
   Then by Postulate 5, the two lines meet and are not parallel.
   Therefore, $\alpha = \beta$.

2. Claim: The exterior angle equals the interior opposite angle ($\delta = \beta$).
   We have that $\delta = \alpha$. (prop. I.15)
   But we have established in part 1, that $\alpha = \beta$.
   Therefore, $\delta = \beta$. (c.n.1)

3. Claim: The interior angles on the same side equal $\varnothing$ ($\gamma + \beta = \varnothing$).
   We have that $\delta + \gamma = \varnothing$. (prop. I.13)
   So substituting $\delta = \beta$ (from part 2), we have,
   $\gamma + \beta = \varnothing$.

Q.E.D.
Comments: 1. This is the first time that Postulate 5 is used in *The Elements*. Because of the wording of Postulate 5, there is speculation that it was written specifically for use in proving Proposition 1.29.

2. In Part 1 of the proof, when we suppose that $\alpha \neq \beta$, we instead suppose, without loss of generality, that $\alpha > \beta$. The same argument can apply if we suppose $\beta > \alpha$. 