Proposition 1.34

In a parallelogram the opposite sides and angles are equal and a diameter bisects the area.

Let $ABDC$ be a parallelogram. We must show that $AB = CD$, $AC = BD$, $\angle CAB = \angle BDC$ and $\angle ACD = \angle ABD$. Since $AC \parallel BD$ and $AB \parallel CD$ we have $\alpha = \gamma$ and $\delta = \beta$ by [I.29]. Hence $\triangle ABC \sim \triangle DCB$, by [I.26]. So we have equality of corresponding sides: $AB = CD$, $AC = BD$, as well as the remaining angles: $\angle CAB = \angle BDC$. Finally, $\angle ACD = \alpha + \beta$ and $\angle ABD = \gamma + \delta$. Since $\alpha = \gamma$ and $\delta = \beta$, we have $\alpha + \beta = \gamma + \delta$, so $\angle ACD = \angle ABD$ by [c.n.2].

Q.E.D.