Side - Angle - Side

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Proposition I.4

If two sides on one triangle are equal (respectively) to two sides on a second triangle and the angles formed by these two sets of lines are equal (respectively) then the bases are equal to each other, the triangles are equal and the remaining angles are equal.
Take two triangles $ABC$ and $DEF$.

Let $AB = DE$ and $AC = DF$ and angle $BAC$ equal angle $EDF$.

Place point $A$ on point $D$ and place $AB$ on $DE$.

Since $AB = DE$, $B$ coincides with $E$.

Because angle $BAC$ equals angle $EDF$, $AC$ coincides with $DF$.

Since $AC = DF$, $C$ coincides with $F$.

Since $B$ coincides with $E$ and $C$ coincides with $F$, $BC$ coincides with $EF$

$BC = EF$ (c.n. 4)

Since all three sides are equal, the triangle $ABC$ coincides with triangle $DEF$.

Triangle $ABC$ is equal to triangle $DEF$. (c.n. 4)

Angle $ABC$ coincides with angle $DEF$, so angle $ABC$ is equal to angle $DEF$.

(c.n. 4)

Angle $ACB$ coincides with angle $DFE$, so angle $ACD$ is equal to angle $DFE$.

(c.n. 4)

Q.E.D

**Comments:**

1. Nothing is used in this proposition except common notion 4, which states that 'things which coincide with one another are equal to one another.' He never explains what he means by 'coincide,' but essentially, he wants to superposition one object onto another, a somewhat shaky proposal.

2. People were actually so unhappy with this proof that a modern mathematician actually turned it into an axiom.

3. In his actual wording, Euclid says 'If one triangle is applied to the other'... it seems to instruct you to put one on top of the other, but that only works if they are oriented in the same direction. If they aren’t, then you have to flip one triangle, meaning we’d be in 3 dimensions and out of the plane.