Proposition III.22

The opposite angles of a quadrilateral in a circle are equal to two right angles.

Claim: \( \angle ADC + \angle ABC = \perp \perp \) and \( \angle DAB + \angle DCB = \perp \perp \)

Draw AC

In \( \triangle ABC \),
\[
\alpha + \beta + \angle ABC = \perp \perp. \quad [I.32]
\]
\[
\alpha = \delta \quad [III.21]
\]
\[
\gamma = \beta \quad [III.21]
\]
\[
\angle ADC = \delta + \gamma
\]
\[
\angle ADC = \alpha + \beta
\]
\[
\angle ADC + \angle ABC = \alpha + \beta + \angle ABC
\]
\[
\angle ADC + \angle ABC = \perp \perp.
\]
Similarly, $\angle DAB + \angle DCB = \perp \perp$.
Q.E.D.

**Comments:**
This proposition does not hold for quadrilaterals not in a circle.

In a three point figure (i.e. a triangle) on a plane, the sum of the angles is always equal to two right angles. This is because for any three noncollinear points there exists a circle, which contains these three points on its circumference.

For a four point figure, however, all four points do not always fall on a circle. If all four points are on the circle, then this quadrilateral is called a **cyclic quadrilateral** and the above proposition proves that the opposite angles will be right angles.