Proposition III.24

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Similar segments of circles on equal straight lines equal one another.

We know that $AB = CD$.
Apply $CFD$ to $AEB$, placing $A$ on $C$, and $B$ on $D$.
Segment $AEB$ must coincide with $CFD$.
If not, then one of two things must be true.
1. $CDF$ is within/outside $AEB$.
However, this is a contradiction because two unequal circle segments cannot be placed on the same straight line. (III.23)
2. $CDF$ falls awry as $AGB$.
But this means a circle cuts another circle at more points than two, which is also a contradiction. (III.10)
Therefore, $AEB$ coincides with and equals $CDF$.
Q.E.D.
Comments:
Given a rectilineal angle and a straight line we can construct a segment with that angle in it.
This is done by making a triangle with that meeting the segment, and then circumscribing a circle around the triangle.
Thus the segment (shown in the picture as $BDE$) has been created.

Q.E.F.