MT 453 Elements

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**Proposition III.37**

*If a point $D$ falls outside a circle $ABC$ and $(DC)(DA) = (DB)^2$, then $DB$ is tangent to the circle.*

Given the circle $ABC$ and the point $D$ outside the circle, draw the secant line $DCA$, where $C$ is the first intersection of the secant and the circle. Connect $DB$, and assume that $(DC)(DA) = (DB)^2$.

From point $D$, draw $DE$ tangent to circle $ABC$. [Prop. III.17]

Then $(DE)^2 = (DC)(DA)$ [Prop. III.36],

So $(DE)^2 = (DB)^2$ and $DE = DB$ [C.N.1]

Find the center, $F$, of circle $ABC$ [Prop. III.1], and connect $FB$, $FD$, and $FE$.

$FE = FB$, since they are both radii, $FD$ is common, and $DB = DE$. 


Therefore $\triangle DFE \cong \triangle DFB$, so $\angle DEF = \angle DBF$. [Prop. I.8]
But $\angle DEF = \angle$ [Prop. III.18], so $\angle DBF = \angle$.
Therefore $DB$ must be a tangent line to circle $ABC$. [Prop. III.16, Por.]

Q.E.D.

Comments:
Euclid also makes the point that this same proof works whether or not $AC$ is a diameter of circle $ABC$. 