Proposition V.22

If there are any number of magnitudes whatever, and others equal to them in multitude, which taken two and two together are in the same ratio, then they are also in the same ratio ex aequali.
Let there be any number of magnitudes $A$, $B$, and $C$, and others $D$, $E$, and $F$ equal to them in multitude, which taken two and two together are in the same ratio, so that $A : B = D : E$, and $B : C = E : F$.

I claim that $A : C = D : F$.

Take equimultiples $G$ and $H$ of $A$ and $D$, equimultiples $K$ and $L$ of $B$ and $E$, and equimultiples $M$ and $N$ of $C$ and $F$.

Since $A : B = D : E$ and equimultiples of those four have been taken, therefore $G : K = H : L$. (V.4)

Similarly, $K : M = L : N$.

If $G > M$, then $H > N$. If $G < M$, then $H < N$. If $G = M$, then $H = N$. (V.20)

And since $G$ and $H$ are equimultiples of $A$ and $D$, and $M$ and $N$ are equimultiples of $C$ and $F$, $A : C = D : F$. (Def V.5)

Q.E.D.