Proposition VI.3

*If an angle in a triangle is bisected the segments of the base are in the same ratio as the remaining sides, and the converse of this statement.*

Draw triangle $ABC$ and bisect $\angle BAC$ with line $AD$ so that point $D$ lies on $BC$. 
Given that \( \angle BAD = \angle DAC \), prove \( BD : DC = BA : CA \).
Draw a line parallel to \( AD \) through point \( C \) [I.31], and extend \( BA \) until it hits that line at point \( E \) [Post 2].
\( \angle DAC = \angle ACE \) [I.29]
\( \angle BAD = \angle BEC \) [I.29]
So \( \angle ACE = \angle BEC \) [c.n.1]
Thus \( AC = AE \) [I.6]
So \( BD : BC = BA : AE \) [VI.2]
Since \( AC = AE, BD : BC = BA : AC \) [c.n.1]

The Converse:
Given \( BD : DC = BA : AC \), prove \( \angle BAD = \angle DAC \).
\( \angle DAC = \angle ACE \) and \( \angle BAD = \angle AEC \) [I.29]
\( BD : DC = BA : AE \) [VI.2]
Using the given information, \( BA : AC = BA : AE \) [V.11]
So \( AC = AE \) [V.9]
So \( \angle ACE = \angle AEC \) [I.5]
Thus \( \angle BAD = \angle DAC \) [c.n.1]. Therefore \( AD \) bisects \( \angle BAC \).
QED