Proposition VI.5

If two triangles have their sides proportional then they will be equiangluar.

On \( \angle FEG = \angle ABC \) and \( \angle EFG = \angle ACB \). [I.23]
This implies \( \angle EGF = \angle BAC \). [I.32]
Therefore \( \triangle ABC \) and \( \triangle EGF \) are equiangular.
\[
\frac{c}{a} = \frac{h}{d}, \ [VI.4]
\text{ but } \frac{c}{a} = \frac{f}{d}. \\
\text{ Thus } \frac{h}{d} = \frac{f}{d}, \ [V.11]
\text{ which means } h = f. \ [V.9]
\[
\frac{b}{a} = \frac{g}{d}, \text{ [VI.4]}
\]
but \( \frac{b}{a} = \frac{e}{d} \).

Thus \( \frac{g}{d} = \frac{e}{d} \), [V.11]

which means \( g = e \). [V.9]

Therefore \( \triangle DEF \simeq \triangle GEF \), [I.8]

since \( h = f \), \( g = e \), and \( d \) common.

So \( \angle DEF = \angle GEF \), \( \angle DFE = \angle GFE \), \( \angle EDF = \angle EGF \),

but \( \angle DEF = \angle ABC \), \( \angle DFE = \angle ACB \), \( \angle EDF = \angle BAC \).

Therefore \( \triangle ABC \) and \( \triangle DEF \) are equiangular.

Q.E.D.