Proposition X.1

Let $\epsilon > 0$. Continually removing more than half of a magnitude leaves a magnitude less that $\epsilon$.

Let $AB$ and $C$ be our magnitudes with $AB > C$. $C$ is out $\epsilon$.

By (def. V.4), $n(C) > AB$.

Let this be $DE$.

Let $n = 3$ so $DF = FG = GE = C$.

Cut off $HB$ on $AB$ so that $HB$ is more than half of $AB$.

Repeat - cut off $HK$ on $AH$ so that $HK$ is more than half of $AH$.

Do this $n$ times.

Claim: $AK < C$

Since $DE = 3(C)$ and we know $C = DE$, we can say $GE < \frac{1}{2}DE$.

$HB > \frac{1}{2}AB$ (by the way we cut it)

$DE > AB$ so $DG > AH$ which we get by taking away $HB$ and $GE$.

Therefore, $\frac{1}{2}DE > \frac{1}{2}AB$ since we took away less than half of $DE$ and more than half of $AB$.

$FG = \frac{1}{2}DG$ and $HK > \frac{1}{2}AH$

Therefore, $DF > AK$. 

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We know $DF = C$, so $C > AK$.

Since $C$ is our $\epsilon$, we can say that $\epsilon > AK$.

Q.E.F.

**Comment:**
This works for any $n$.

**Porism:**
This works for removing exactly half as well.