Proposition XII.2

Circles are to one another as the squares on their diameters
Let $\odot ABCD$ and $\odot EFGH$ be circles with diameters $BD$ and $FG$, respectively. Let $S$ be a magnitude such that $BD^2 : FG^2 = \odot ABCD : S$. I claim that $S = \odot EFGH$.
Suppose $S < \odot EFGH$.
Inscribe a square $\Box EFGH$ in the circle $\odot EFGH$; then $\Box EFGH$ is greater than half of $\odot EFGH$.
Bisect the arcs $EF, FH, HG, GE$ at $K, L, M, N$ respectively.

Then $\triangle EKF$ is greater than half of arc $EF$, etc.
Continue bisecting until the remaining segments are less than $S - \odot EFGH$. [X.1]
This gives a polygon $[E]$ with diameter $FG$ which is greater than $S$.
Inscribe a polygon $[A]$, with diameter $BD$, and similar to $[E]$, in $\odot ABCD$.

Then $BD^2 : FG^2 = [A] : [E]$, [XII.1]
But $[E] > S$, so $\odot ABCD : [A]$, which is impossible.
Therefore we cannot have $BD^2 : FG^2 = \odot ABCD : S$ with $S < \odot EFGH$.
Suppose we have $BD^2 : FG^2 = \odot ABCD : S$ with $S > \odot EFGH$.
Then $FG^2 : BD^2 = S : \odot ABCD = \odot EFGH : T$, where $T < \odot ABCD$.
This too is impossible, by the the argument of the first part, with the circles interchanged.
Therefore we have $BD^2 : FG^2 = \odot ABCD : \odot EFGH$, so that the circles are as the squares on their diameters.
Q.E.D.