Notes 3,4,5 EXTRA PROBLEMS

1. Compute the double integrals \( \iint_R f \, dR \), where \( R \) is the square \( 0 \leq x \leq \pi, \ 0 \leq y \leq \pi \), for the following functions \( f(x,y) \)
   a) \( f(x,y) = xy \)
   b) \( f(x,y) = x^3 - 3xy^2 \cos y \).
   c) \( f(x,y) = \sinh(x) \cos(y) \)

2. Let \( R \) be the disk of radius \( a \) centered at \((x_0, y_0)\). Compute \( \iint_R f \, dR \) for the following functions \( f(x,y) \).
   a) \( f(x,y) = x^2 \).
   b) \( f(x,y) = xy \).
   c) \( f(x,y) = x^4 - 6x^2y^2 \) (Use the Jacobian and exercise 3.5 in Note 4.)

3. Compute
\[
\iint_R e^{-x^2 - y^2} \, dR
\]
where \( R \) is the disk of radius \( a \) centered at \((0,0)\).

4. Compute the average of \( x^{2n}y^{2m} \) over the unit circle. (This is very easy, if you think about the double integral over the unit disk, which you have already done.)

5. Compute the integral of \( f(x,y) = x + 2y - 3 \) over the square \( R \) with vertices \((1,1), \ (2,2), \ (1,3), \ (0,2)\), in THREE ways:
   i) Find the area and center of mass.
   ii) Find \( P \) and \( Q \) so that \( Q_x - P_y = x + 2y - 3 \), and use Green's theorem.
   iii) Find a map \( R(u,v) \) from the nice square \( 0 \leq x, y \leq 1 \) to \( R \) and use the Jacobian.

6. Compute the integral of \( x^{100} \) over the unit disk centered at \((0,0)\).

7. Find the measures of the spheres \( S^{n-1} \) for \( 1 \leq n \leq 10 \).

8. Calculate the derivative with respect to \( a \) of the measure of the ball \( B^n(a) \). Then set \( a = 1 \). What do you get? (If you want, just do it for \( 1 \leq n \leq 10 \).)

9. Use Green's Theorem to compute the line integral \( \int_c y \, dx + x \, dy \), where \( c \) is the counterclockwise hexagon whose six vertices are \((\cos \frac{2k\pi}{6}, \sin \frac{2k\pi}{6})\), \( k = 0,1,2,3,4,5 \). (You'll need to find the area of the hexagon. A hexagon is made of triangles. We know the area of a triangle (extra problems for Note 2).)

10. Use line integrals to find the center of mass of the triangle with vertices \((0,0), \ (a,b), \ (c,d)\).

11. Here is a new mapping to think about.
\[
R(u,v) = (u \cosh v, u \sinh v).
\]

It is the hyperbolic analogue of polar coordinates. Find the Jacobian of \( R(u,v) \). Draw the images under \( R(u,v) \) of horizontal and vertical lines in the \( u,v \) plane. Just consider \( 0 \leq u \leq 1, \ 0 \leq v \leq \infty \).